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## PHASE RADIATION CHARACTERISTICS OF AN OPEN-ENDED CIRCULAR WAVEGUIDE

A.V. Shishkova<sup>1</sup>, S.N. Pivnenko<sup>2</sup>, O.S. Kim<sup>2</sup>, N.N. Gorobets<sup>1</sup>

<sup>1</sup> Kharkov National University, 4 Svobody sq., Kharkov, 61077, Ukraine  
Phone: +380-572-434525, fax: +380-572-471816, e-mail: shann@online.kharkiv.com

<sup>2</sup> Technical University of Denmark, Oersteds Plads, 348, 2800 Kgs. Lyngby, Denmark  
Phone: +45-4525-3860, fax: +45-4593-1634, e-mail: sp@oersted.dtu.dk

### ABSTRACT

Analytic expressions for phase radiation characteristics of a semi-infinite open-ended circular waveguide regardless of its aperture size and operating frequency have been obtained making use of the rigorous Weinstein's theory. The analysis of phase radiation patterns has been carried out for the dominant mode (TE<sub>11</sub>) as well as for the high order modes TM<sub>01</sub> and TE<sub>01</sub>, both for a single and multimode propagation. The measurement of radiation characteristics of an open-ended circular waveguide has been carried out at the DTU-ESA Spherical Near-Field Antenna Test Facility. It is shown that the theoretical results are in a good agreement with the experimental ones.

### ANALYTIC EXPRESSION FOR PHASE RADIATION PATTERN

For the first time, a rigorous solution of the electromagnetic diffraction problem for an open-ended circular waveguide (OE-CWG) by the Wiener-Hopf method was proposed by L. A. Weinstein [1]. He obtained analytic expressions and carried out a thorough analysis of amplitude radiation patterns for the case of single mode diffraction at an OE-CWG. When an open-ended waveguide is used as a feed in an antenna system, both the amplitude and phase radiation characteristics are needed thus allowing the polarization characteristics of the feed to be calculated. In this paper, the analytic expressions for the phase radiation characteristics of a semi-infinite OE-CWG are obtained regardless of its aperture size and operating frequency. According to the Weinstein's theory the explicit expression for the amplitude radiation pattern contains finite products proportional to the number of high order modes propagating in the waveguide of the given size. The phase radiation pattern, in general, contains an infinite sum, such as:

$$K = \lim_{M \rightarrow \infty} \frac{2w_0}{\pi} \left[ \ln \frac{2M\pi}{v_1 + \sqrt{v_1^2 - x^2}} - \frac{\pi}{w_0} \sum_{n=N_r+1}^M \arcsin \frac{w_0}{\sqrt{v_n^2 - v_0^2}} \right], \quad (1)$$

where  $v_n$  is a  $n$ -th root of the Bessel function of the first kind,  $v = \sqrt{k^2 - w^2}$  is a transverse wavenumber, and  $w$  is a longitudinal wavenumber.

In order to obtain an analytic expression for the phase radiation pattern, a method of summation of rational series using poly-gamma functions [2] has been employed. According to this method, there is an explicit expression for the following infinite series:

$$\sum_{n=\tilde{N}}^{\infty} \left( \frac{1}{n} - \frac{1}{n+1/4} \right) = \left( \Psi \left( \tilde{N} + \frac{1}{4} \right) - \Psi(\tilde{N}) \right), \quad (2)$$

where  $\Psi$  is the di-gamma function [2]. Employing asymptotic expressions for the roots of the Bessel function and expanding arccosine function in a series for a small value of argument, the infinite series in (1) can be reduced to the summation (2). The similar technique has been employed to obtain analytic expressions for the phase radiation pattern of an OE-CWG for both the symmetric and the non-symmetric excitation modes.

Investigation of the phase behavior on a whole radiation sphere has shown that the phase radiation patterns are not uniform for the symmetric excitation modes  $TM_{01}$  and  $TE_{01}$  as well as for the dominant mode  $TE_{11}$ . At the same time, it is well known that calculation of the phase radiation characteristics for a circular aperture under the Kirchhoff approximation without taking into account phenomena associated with mode transformation at the aperture and neglecting currents flowing on an exterior surface gives a fictitious phase center located in the center of the aperture [3]. Therefore, accounting for mode diffraction at the open end of a waveguide leads to the conclusion that a radiator in the form of OE-CWG has no phase center regardless of the excitation mode. However, when an OECWG is used as a feed in reflector or lens antennas, the phase error over the aperture associated with the non-uniform phase radiation pattern is quite small. Thus for the dominant mode excitation ( $TE_{11}$ ), the deviation of the phase pattern within the main lobe at  $-3$  dB does not exceed 2 degrees as compared to the direction of the main radiation maximum (See Fig. 1a). This remains valid for the whole operating frequency range of the single mode waveguide:  $1.84 < ka < 3.83$ , where  $k$  is a longitudinal wavenumber in the free space and  $a$  is the radius of the waveguide. For the

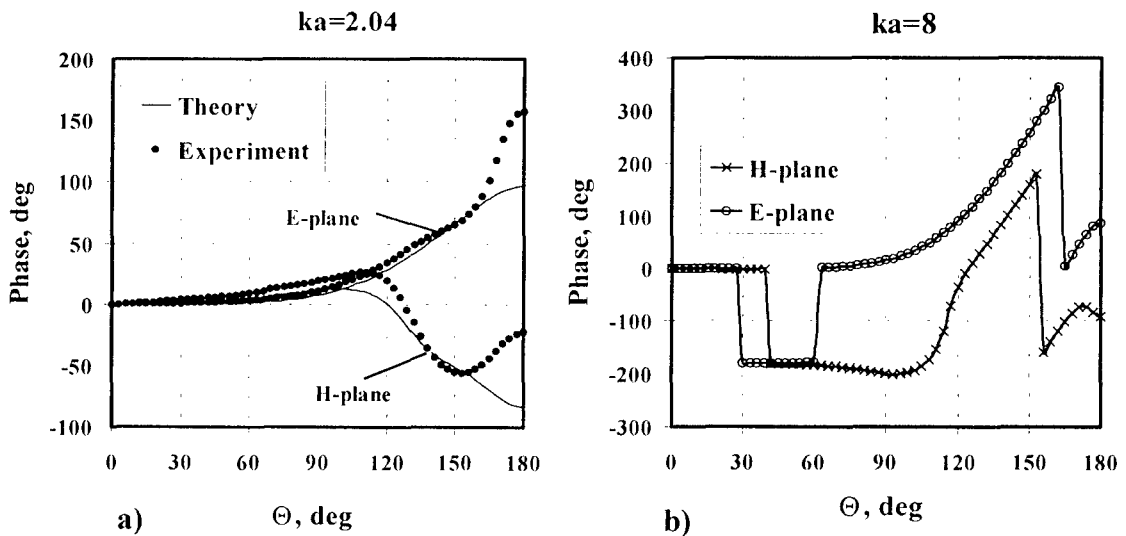


Fig.1. Phase radiation pattern of the single mode (a) and the multimode waveguide (b).

case of  $TE_{01}$  excitation ( $4.75 < ka < 6.5$ ), the deviation within the main lobe does not exceed 4 degrees, and for the  $TM_{01}$  case ( $3.0 < ka < 5.0$ ), it does not exceed 5 degrees. The maximum deviation of the phase radiation pattern within the main lobe is observed near cut-off frequencies: for  $TE_{01}$  mode with  $ka = 4.0$  it runs up to  $-21$  degrees and for  $TM_{01}$  mode with  $ka = 2.5$  it consists of 15 degrees. In the whole forward hemisphere the deviation of the phase radiation pattern can reach 30 degrees. The phase radiation pattern of an oversized waveguide can be considered almost uniform both within the main lobe and within first sidelobes (Fig. 1b).

## NEAR-FIELD PHASE RADIATION CHARACTERISTICS

The spherical wave expansion technique is used to analyze the general *near-field* radiation characteristics of an OE-CWG excited by the dominant mode  $TE_{11}$  as well as by the higher-order modes  $TE_{01}$  and  $TM_{01}$ . First, the coefficients of the spherical-wave expansion are obtained by matching the expansion with the far-fields. Then, the coefficients are used to calculate the near field.

An experimental verification of the calculated amplitude and phase radiation patterns has been made. The measurement of radiation characteristics of an open-ended circular waveguide has been carried out at the DTU-ESA Spherical Near-Field Antenna Test Facility. The measurement was performed at several frequencies for the dominant mode  $TE_{11}$  and for the  $TM_{01}$  mode as well. Two orthogonal complex components of the radiated field were accurately measured on a full sphere around the open-ended waveguide by a dual polarized probe. The measurement data were then transformed both to the far-field and to the near-field [4]. The theoretical amplitude and phase radiation characteristics of the waveguide were compared to the results obtained from the measurements (see Fig. 1a). It is seen that the theoretical results are in a good agreement with the experimental ones. Some differences yet observed can be explained by the difference between the simulated *semi-infinite* waveguide and the measured *finite* waveguide.

## CONCLUSION

The investigation of the phase radiation characteristics of an open-ended circular waveguide has shown that it has no phase center regardless of the excitation mode. Maximum deviation of the phase pattern from a constant is observed for the single mode propagation. In the multimode operation the phase radiation pattern is nearly uniform within the main lobe. In the near-field the phase radiation pattern is not uniform both for the single and multimode waveguides. As the distance to the aperture decreases, the deviation of the phase radiation pattern becomes more pronounced.

## REFERENCES

- [1] L.A. Weinstein. *The theory of diffraction and the factorization method*, The Boleyn Press, Boulder, Colorado, 1969.
- [2] M. Abramovits and I. A. Stegun. *Handbook of mathematical functions*, Appl. Mathem. series-55, 1964, pp. 84-90.
- [3] Yu.V. Shubarin. *Microwave antennas*, Izdat. Kharkov. Universiteta, 1960. (in Russian)
- [4] J.E. Hansen. *Spherical near-field antenna measurements*, London, Peter Peregrinus, 1988.